V. SIMULATION AND CONTROL

M. I. Sobhy, Applications of Padé approximants in electrical network problems

J. B. Knowles, A. B. Keats and D. W. Leggett, The simulation of a continuously variable transport delay

Y. Shamash, Approximation of linear time-invariant systems

S. C. Chuang, Frequency domain approximation technique for optimal control

H. P. Debart, A Padé Chebyshev approximation in network theory

The reader interested in the applications of Padé approximation to theoretical physics may also wish to consult [1], which contains another recent cross-section of work in this field.

W. G.

1. G. A. BAKER, JR. & J. L. GAMMEL, Editors, *The Padé Approximant in Theoretical Physics*, Academic Press, New York and London, 1970.

31 [2.05, 2.35, 3.25, 13.35].—L. COLLATZ & W. WETTERLING, Editors, *Numerische Methoden bei Optimierungsaufgaben* (in German and English), International Series of Numerical Mathematics, vol. 17, Birkhäuser Verlag, Basel, 1973, 136 pp., 25 cm. Price: approximately $10.—.

Proceedings of a conference on Numerical Methods for Optimization Problems which was held November 14–20, 1971, in Oberwolfach, Germany. In addition to the papers appearing in the book, the following are given in the foreword as the most important points raised in discussions among the participants:

1. Many of the familiar methods used in optimization problems, being frequently the developments of those not oriented toward numerical analysis or computation, must be more closely examined than they have been in the past for their numerical fitness, and improved, if necessary.

2. In iterative methods, the determination of an initial approximation is frequently more difficult than the execution of the method itself. This should be taken into consideration when numerical algorithms are developed.

3. Several new (asymptotic) methods for integer programming have appeared. Still, the standard difficulty remains that their computational complexity is not constrained by a bound depending only upon the dimension of the problem.

4. New numerical methods which have been developed at universities are frequently unusable for the practitioner because the originators have not tested their methods sufficiently on applications problems and, consequently, cannot give adequate directions for their employment. This deficiency has to be remedied.

Papers presented were:

1. Bereanu, B.: "The Cartesian Integration Method in Stochastic Linear Programming."

A computational method for finding the expected value and the probability-distribution function for the optima of stochastic linear programs is proposed when the constraint matrix, right-hand side, and cost vector are affine functions of a random vector sampled from an r-dimensional compact interval and when certain other assumptions hold.

2. Collatz, L.: "Anwendungen der Dualitaet der Optimierungstheorie auf nichtlineare Approximationsaufgaben [Applications of Duality in Optimization Theory to Nonlinear Approximation Problems]."

Given the space $C(B)$ of continuous functions over $B \subseteq R^n$ (compact), some $f \in C(B)$, and a subspace of functions $w(x,\alpha)$ in $C(B)$ which depend upon parameters $\alpha = (\alpha_1, \cdots, \alpha_p)$, consider the primal problem: α, δ_1, δ_2 so that $-\delta_1 \leqq w(x,\alpha) \leqq \delta_2$ for all x in B and $\delta_1 + \delta_2$ is minimal. This problem generalizes that of Chebyshev approximation. Its dual is: find linear, bounded, nonnegative functionals l_1, l_2 on $C(B)$ so that $l_2(w) - l_1(w)$ is nonnegative for all $w(x,\alpha)$, so that $l_2(1) = l_1(1) = 1$, and so that $l_1(f) - l_2(f)$ is maximal. The application of a weak duality theorem to obtain bounds on $\delta_1 + \delta_2$ is described via some simple examples.

3. Eckhardt, U.: "Iterative Loesung linearer Ungleichungssysteme [Iterative Solution of Linear Inequality Systems]."

For M a given subset of a Hilbert space H, K the convex hull of M, and b a given point in H, an algorithm is sketched which attempts to generate a sequence in K converging to b. Should b not lie in the closure of K, a point $\hat{x}$ is produced so that the inner product $\langle \hat{x}, y - b \rangle$ is positive for all y in M. This algorithm is applied to get derivative-free truncation-error expressions for numerical quadrature formulas with positive weights. The algorithm is imprecisely defined in the paper, but reference is given to a technical report which contains further details.

4. Fleischmann, B.: "Eine primale Version des Benders'schen Dekompositionsverfahrens und seine Anwendung in der gemischt-ganzzahligen Optimierung [A Primal Version of Benders' Decomposition Algorithm and Its Application in Mixed Integer Programming]."

The problem in question is

$$C^T x + f(y) = \max, \qquad Ax + F(y) \leqq b, \qquad y \in S.$$

Benders splits this problem into a sequence of alternately linear programming problems involving x and optimization problems over S. These "S-problems" are solved by a dual method. Difficulties with this approach are pointed out, and a modification is given defining the S-problems so that a primal method can reasonably be applied. Some numerical results are presented.

5. Glashoff, K.: "Schwache Stetigkeit bei nichtlinearen Kontrollproblemen [Weak Continuity in Nonlinear Control Problems]."

A class of nonlinear control problems is presented

$$\underset{u \in Q}{\text{minimize}}\, C(u)$$

where Q (the set of permissible control functions) is a subset of a certain Hilbert space and c is a certain type of control functional. The theorem that a functional which is weakly semicontinuous from below on a Hilbert space takes on its minimum over weakly compact subsets of the space is used to give existence results. Conditions on control problems of the given class which guarantee the appropriate compactness and continuity are discussed.

6. Gustafson, S. A.: "Die Berechnung von verallgemeinerten Quadraturformeln vom Gaussschen Typus, eine Optimierungsaufgabe [The Calculation of Generalized Quadrature Formulas of Gaussian Type, an Optimization Problem]."

A number of examples involving the approximation of functionals over a class of absolutely monotone functions are presented. Nonlinear systems are encountered, the solution of which motivates the discussion of an extremal property connected with the moment cone of Chebyshev systems. Some numerical results are presented.

7. Krabs, W.: "Stetigkeitsfragen bei der Diskretisierung konvexer Optimierungsprobleme [Questions of Continuity Connected with the Discretization of Convex Optimization Problems]."

For X a convex subset of a normed linear space E and for Z a normed vector space partially ordered by the cone Y, the problem considered is: find $\inf f(x)$ subject to $x \in X$ and $g(x) \in Y$. Here $f: E \to \mathbf{R}$ is convex and $g: E \to Z$ is concave. General sequences of approximating problems involving sets X_m and functions g_m and f_m are considered, and some results concerning the convergence of the solutions for these problems are presented. A linear control example is investigated in some detail.

8. Kubik, K.: "Das Problem Slalom oder optimale Linienfuehrung innerhalb eines Korridors—ein nichtlineares Optimierungsproblem [The Slalom Problem, or Optimal Curve Fitting Within a Corridor—A Nonlinear Optimization Problem]."

A generalization of interpolation is discussed: the problem of constructing a smooth, low-curvature, minimum-length curve between two points so that it intersects each of the discs in one given collection while avoiding each of those in a second collection. The pragmatics of an optimization-scheme are presented. No theory. Delightfully illustrated.

9. Lempio, F.: "Dualitaet und optimale Steuerungen [Duality and Optimal Control]."

For the most general formulation of an optimization problem, a dual problem is specified and a theorem of weak inclusion stated, all without assumptions about linearity, convexity, or differentiability. An example control problem is posed, its dual is studied, and an upper bound on the solution of the primal problem is obtained from the weak inclusion theorem. Then a strong inclusion theorem is derived in the case of linear optimization.

10. Locher, F.: "Optimale definite Polynome und Quadraturformeln [Optimal Definite Polynomials and Quadrature Formulas]."

A definite polynomial is defined as one which does not change sign on $[-1, +1]$. The problem considered is that of finding the extreme value for $\int_{-1}^{+1} p_n(x) w(x) dx$ with p_n taken from among all definite monic polynomials of degree $\leqq n$ and $w(x)$ a given nonnegative weight function. Application of the solution to this problem is made to determine some quadrature formulas and to solve a problem posed by P. Kirchberger.

11. Sibony, M.: "Some Numerical Techniques for Optimal Control Governed by Partial Differential Equation" (sic).

Considered as problem (1) is: find $u \in X$ so that $F(u) \leqq F(v)$ for all $v \in X$, where X is a closed convex subset of a Banach space and F is a given functional. A reformulation of this with greater detail in the case of control problems governed by partial differential equations is given as problem (2). Several illustrations of obtaining numerical solutions for both problem types are presented.

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32 [2.05.3, 2.20, 2.40].—RICHARD P. BRENT, *Algorithms for Minimization Without Derivatives*, Prentice-Hall, Englewood Cliffs, N. J., 1973, xii + 195 pp., 24 cm. Price \$12.—.

This well written very readable book should be of particular interest to numerical analysts working on methods for finding zeros and extrema of functions. The book is concerned primarily with functions of a single variable and exclusively with methods which use only function values; no derivative evaluations are required by any of the algorithms presented. It also emphasizes algorithmic details that are extremely important for developing reliable computer codes.

In the first chapter, a very useful summary is given of the material covered in subsequent chapters. As these chapters are relatively self-contained, this enables the reader to easily determine which sections of the book to read according to his interests.

Fundamental results on Taylor series, polynomial interpolation, and divided differences are found in Chapter 2. These results are proved under slightly weaker assumptions than one usually finds in the literature. Chapter 3 presents a unified treatment of successive polynomial interpolation for finding zeros of a function and its derivatives; i.e., zeros, stationary points, inflexion points, etc.

The next four chapters are devoted, respectively, to algorithms for: